SSA Register Allocation

Sebastian Hack, Daniel Grund, Gerhard Goos

(hack|ggoos)@ipd.info.uni-karlsruhe.de grund@cs.uni-sb.de

Institut für Programmstrukturen und Datenorganisation
Universität Karlsruhe

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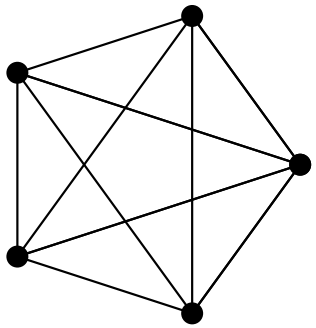
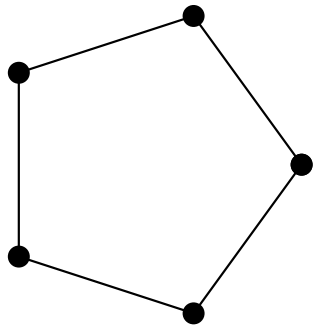
Reading

Sebastian Hack, Daniel Grund, Gerhard Goos
Towards Register Allocation for Programs in SSA-form
Universität Karlsruhe, Technical Report 2005-27, September 2005
`http://www.info.uni-karlsruhe.de/~hack/ra_ssa.pdf`

Complete Graphs and Cycles

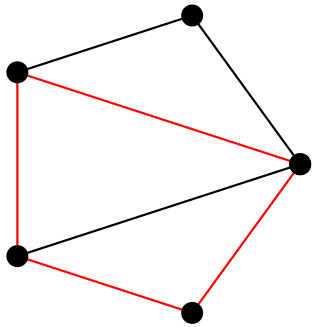


Complete Graph K^5

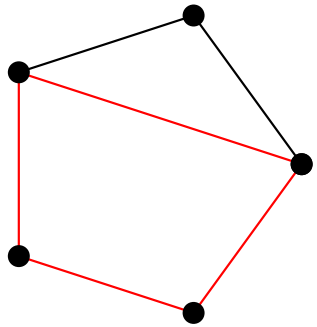


Cycle C^5

Induced Subgraphs



Graph with a C^4 subgraph



Graph with a C^4 induced subgraph

Note

Induced complete graphs are called cliques

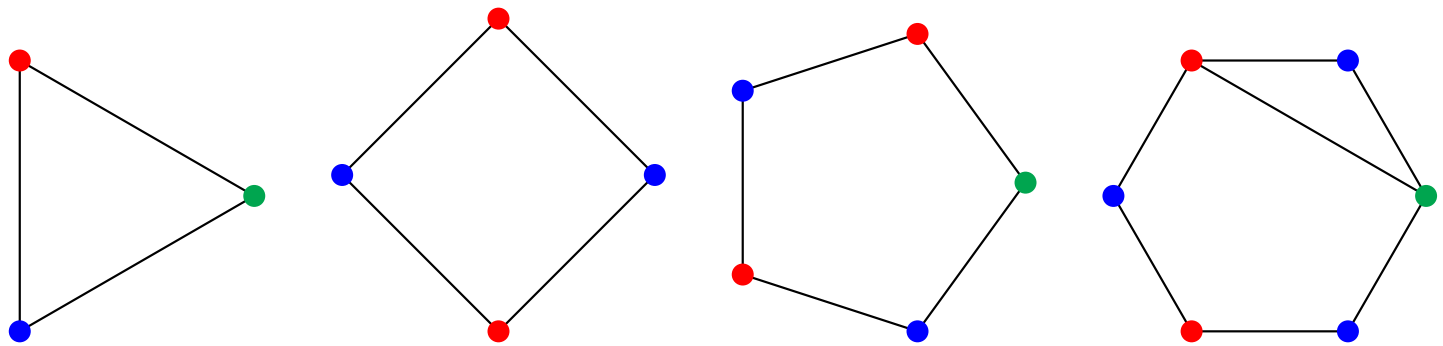
Clique number and Chromatic number

$\omega(G)$ Size of the largest clique in G

$\chi(G)$ Number of colors in a minimum coloring of G

Corollary

$\omega(G) \leq \chi(G)$ *holds for each graph* G

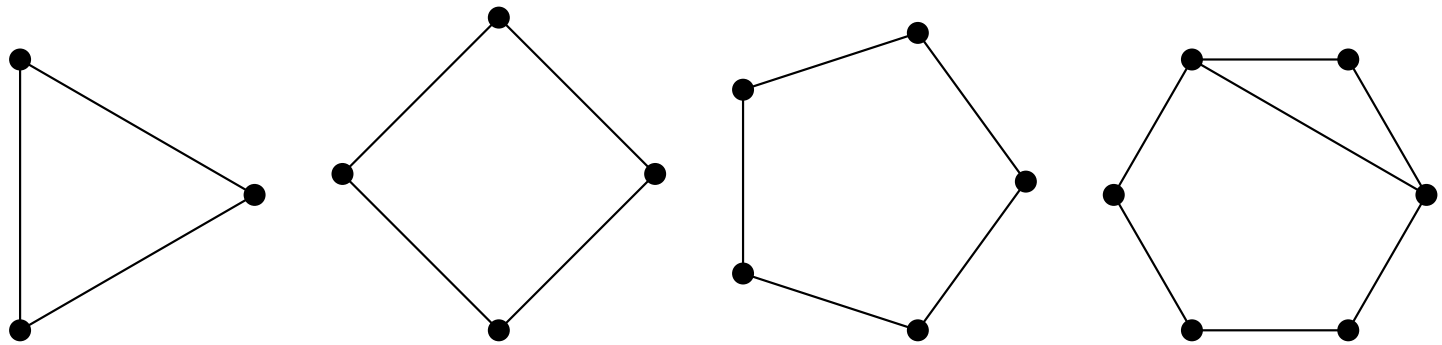


$\omega(G)$	3	2	2	3
$\chi(G)$	3	2	3	3

Perfect Graphs

Definition

G is perfect $\iff$ $\chi(H) = \omega(H)$ for each induced subgraph H of G

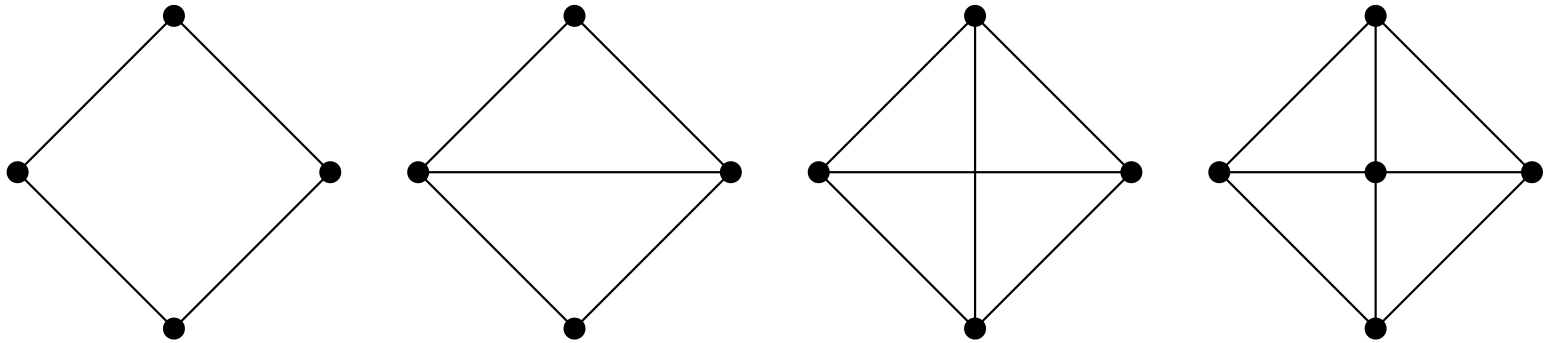


perfect?	✓	✓		

Chordal Graphs

Definition

G is chordal $\iff$ G contains no induced cycles longer than 3



chordal?		✓	✓	

Theorem

Chordal graphs are perfect

Theorem

Chordal graphs can be colored optimally in $O(|V| \cdot \omega(G))$

Register Allocation

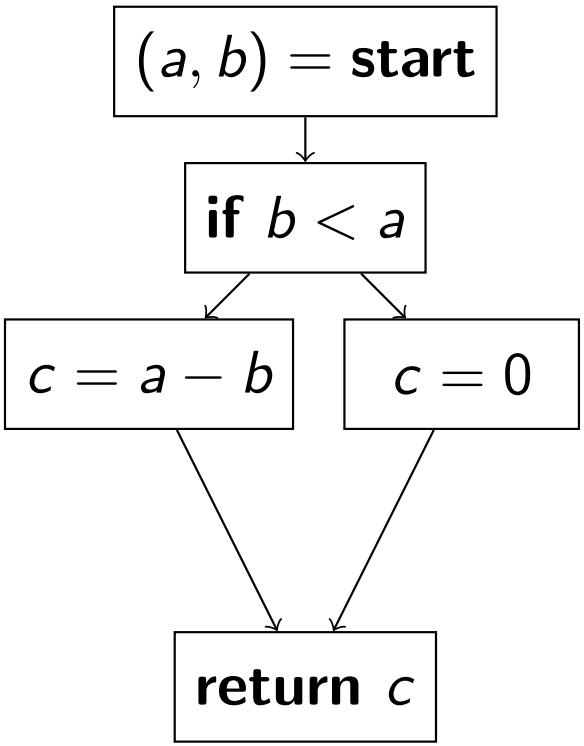
- Register Allocation is the task of mapping the program's variables to processor registers
- Issues to be covered:

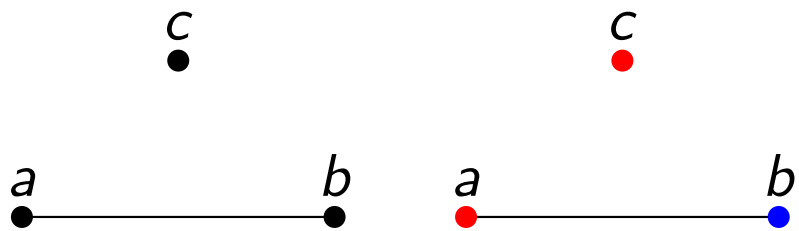
  Spilling Put variables into memory if there are not enough registers

  Coalescing Eliminate unneccessary copies in the program
- Often reduced to graph coloring

Interference Graphs

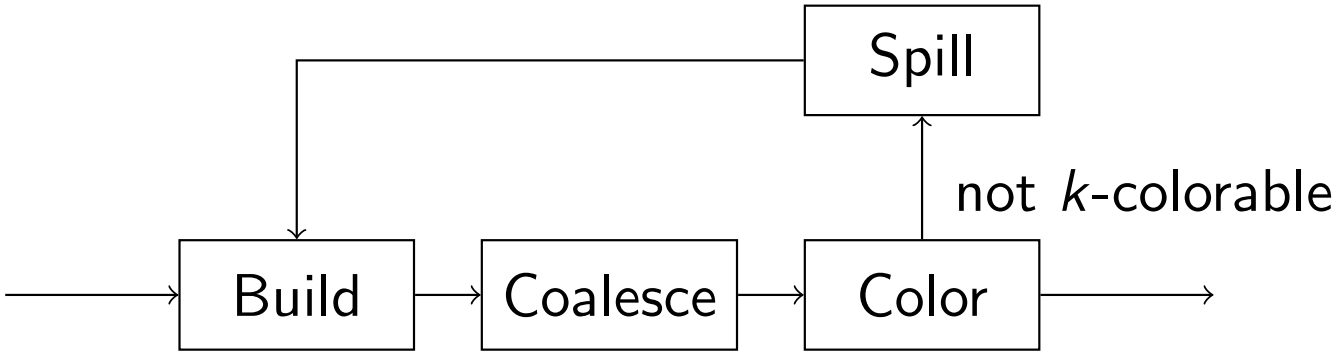
- Two variables are live at the same label ▶ they interfere
- Each variable has a node in the interference graph (IG)
- Whenever two variables interfere, there is an edge between the corresponding nodes





Coloring gives register allocation

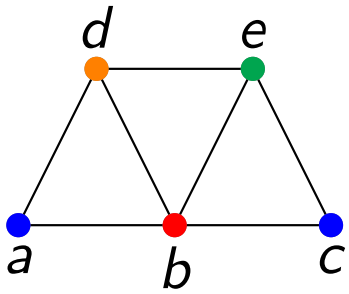
Chaitin/Briggs Register Allocator



- Every undirected graph can occur as an interference graph
- Determining chromatic number is NP-complete
- Color using heuristic $\Rightarrow$ Iteration necessary
- Spilling is focused on the graph

Coloring

- Subsequently remove the nodes from the graph
- Re-insert the nodes in reverse order
- Assign each node the next possible color



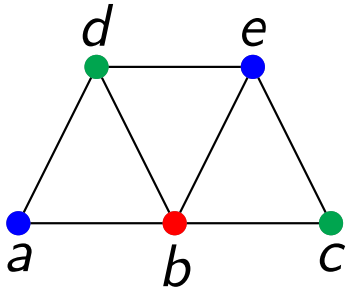
elimination order
d, e, c, a, b

Theorem (from graph theory)

For each graph there is an elimination order leading to an optimal coloring

Perfect Elimination Orders

- Suppose all (not yet eliminated) neighbors of a node *n* form a clique



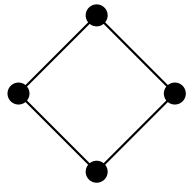
elimination order
a, c, d, e, b

Theorem (from graph theory)

- *A PEO allows for an optimal coloring in polynomial time*
- *The number of colors is bound by the size of the largest clique*

Perfect Elimination Orders

- Graphs with holes larger than 3 have no PEOs, e.g.



- Graphs with PEOs are called chordal
- Chordal graphs are perfect, i.e.

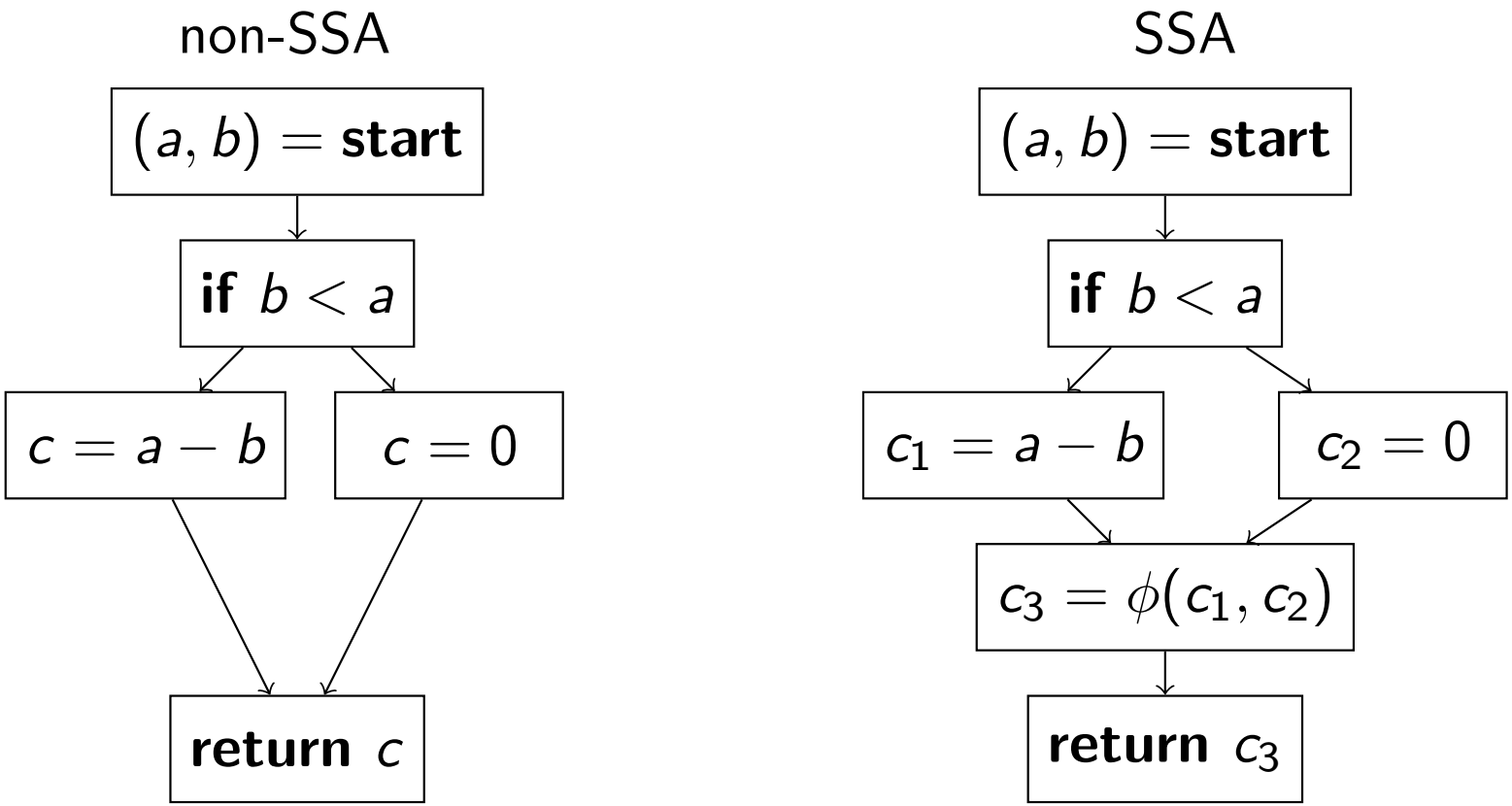
$$\chi(H) = \omega(H) \text{ for each } H \subseteq G$$

Theorem

- The dominance relation in SSA programs induces a PEO in the IG
- Thus, SSA IGs are chordal

SSA-Form

- Each variable has exactly one definition
  - Identity of variables and dynamic constants (values)
- ϕ-operations select values dependent on control flow

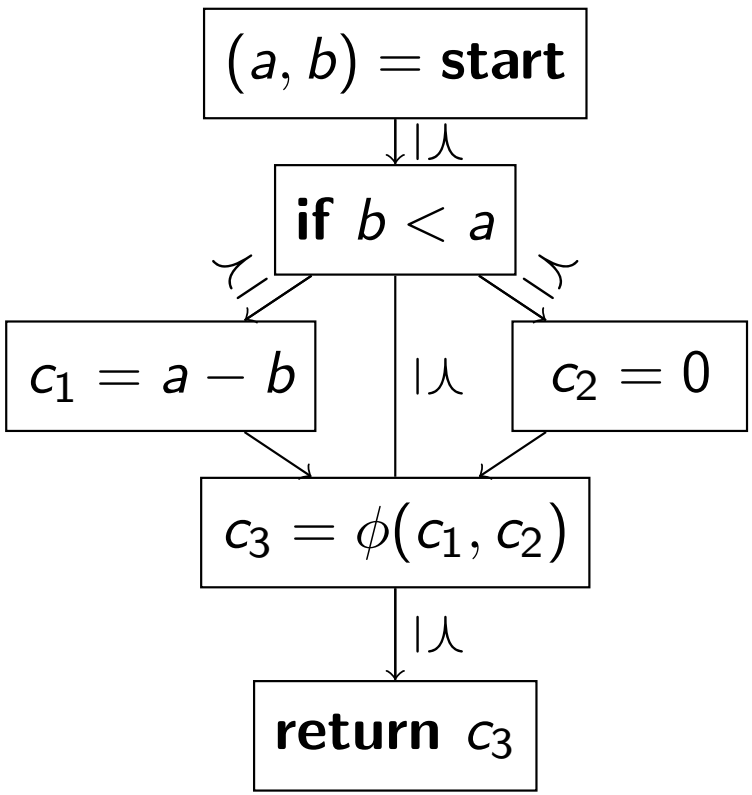


Dominance

Crucial for SSA-form programs is the concept of dominance:

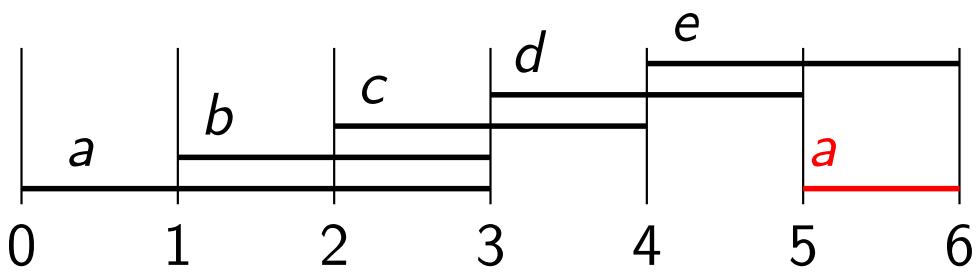
Definition

ℓ_1 dominates ℓ_2 if each path from **start** to ℓ_2 goes through ℓ_1

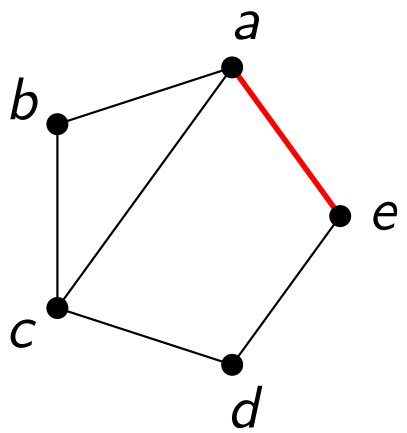


- Each node has a unique immediate dominator
- Thus, dominance induces a tree on the control flow graph
- Thus, dominance is also a partial order

Why are SSA IGs chordal? — Intuition



- Each interval corresponds to a lifetime of a variable
  - a node in the interference graph

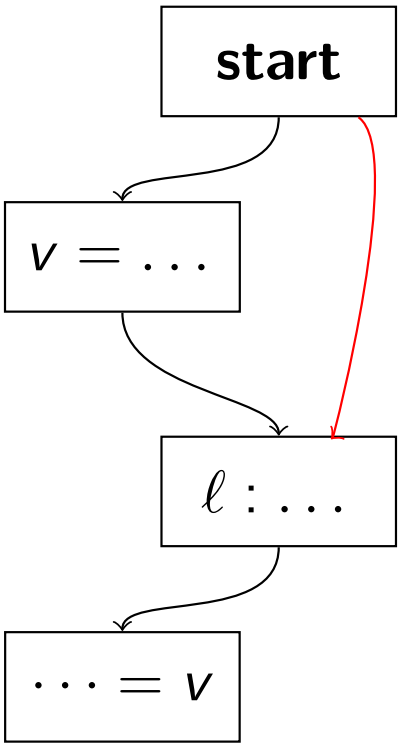


- Can we make a cycle by drawing an edge from *a* to *e*?
- Only by letting *a* "start again" at 5

Liveness and Dominance

Budimlić, PLDI '02

- Each label where a value v is live, is dominated by v's definition.
- We write shortly: $\mathcal{D}_v \preceq \ell$

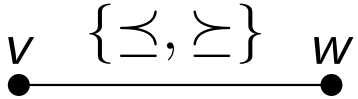


Proof by contradiction

- Assume ℓ is not dominated by $\mathcal{D}_v$
- Then there's a path from **start** to some usage of v not containing the definition of v
- This cannot be since each value must have been defined before it is used

Interference and Dominance I

- Assume v, w interfere, i.e. they are live at some label ℓ
- Then, $\mathcal{D}_v \preceq \ell$ and $\mathcal{D}_w \preceq \ell$
- Since dominance is a tree, either $\mathcal{D}_v \preceq \mathcal{D}_w$ or $\mathcal{D}_w \preceq \mathcal{D}_v$



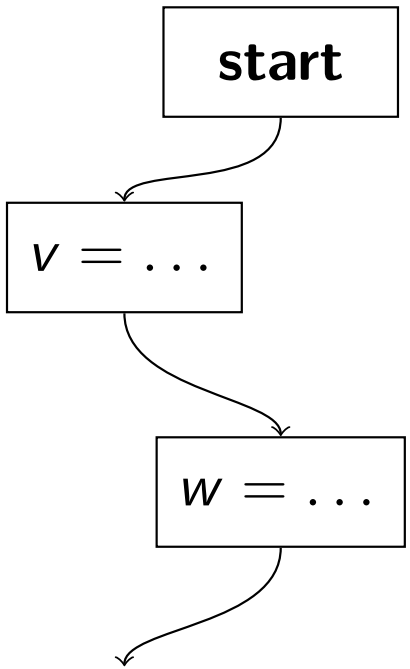
Consequences

- Each edge in the interference graph has a direction according to dominance
- The interference graph is an "excerpt" of the dominance relation

Interference and Dominance II

Budimlić, PLDI '02

- Assume $v \preceq w$
- Then, v is live at $\mathcal{D}_w$

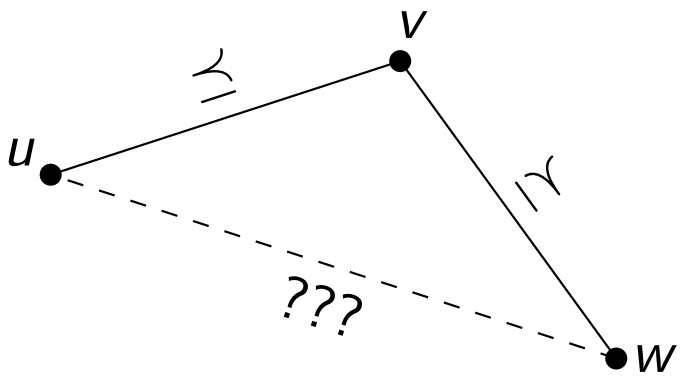


Proof by contradiction

- Assume v was not live at $\mathcal{D}_w$
- Then there is no path from $\mathcal{D}_w$ to any use of v
- So v and w do not interfere $\not\downarrow$

Interference and Dominance III

- Consider three nodes u, v, w in the IG:



- u is live at $\mathcal{D}_v$
- w is live at $\mathcal{D}_v$
- Thus, they interfere

Conclusion

All values

- interfering with v
- whose definitions dominate the one of v

are members of the same clique

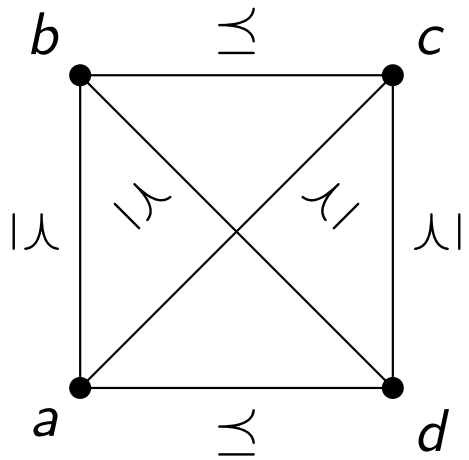
Dominance and PEOs

- Before a value v is added to a PEO,
  add all values whose definitions are dominated by $\mathcal{D}_v$
- Thus, a post order walk of the dominance tree defines a PEO
- IGs of SSA-form programs can be colored in $O(\chi(G) \cdot |V|)$
- Ideally without constructing the interference graph itself

Spilling

Theorem

For each clique in the IG there is a label in the program where all nodes in the clique are live.



- Dominance induces a total order inside the clique
  $\Rightarrow$ There is a "greatest" value d
- All others are live at the definition of d

Spilling

Consequences

- The chromatic number of the IG is exactly determined by the number of live variables at the labels
- Lowering the number of values live at each label to k makes the IG k-colorable
- We know in advance where values must be spilled
  - All labels where the pressure is larger than k
- Spilling can be done before coloring and
- coloring will always succeed afterwards

Conclusion

- No iteration as in Chaitin/Briggs-allocators
- Interference graph has to be built only once (if at all)

Getting out of SSA

- We now have a k-coloring of the SSA interference graph
- Can we turn it into a valid register allocation using k registers for the corresponding non-SSA program?

Central question

How to handle ϕ-functions?

ϕ-Functions

- All ϕ-functions in a basic block

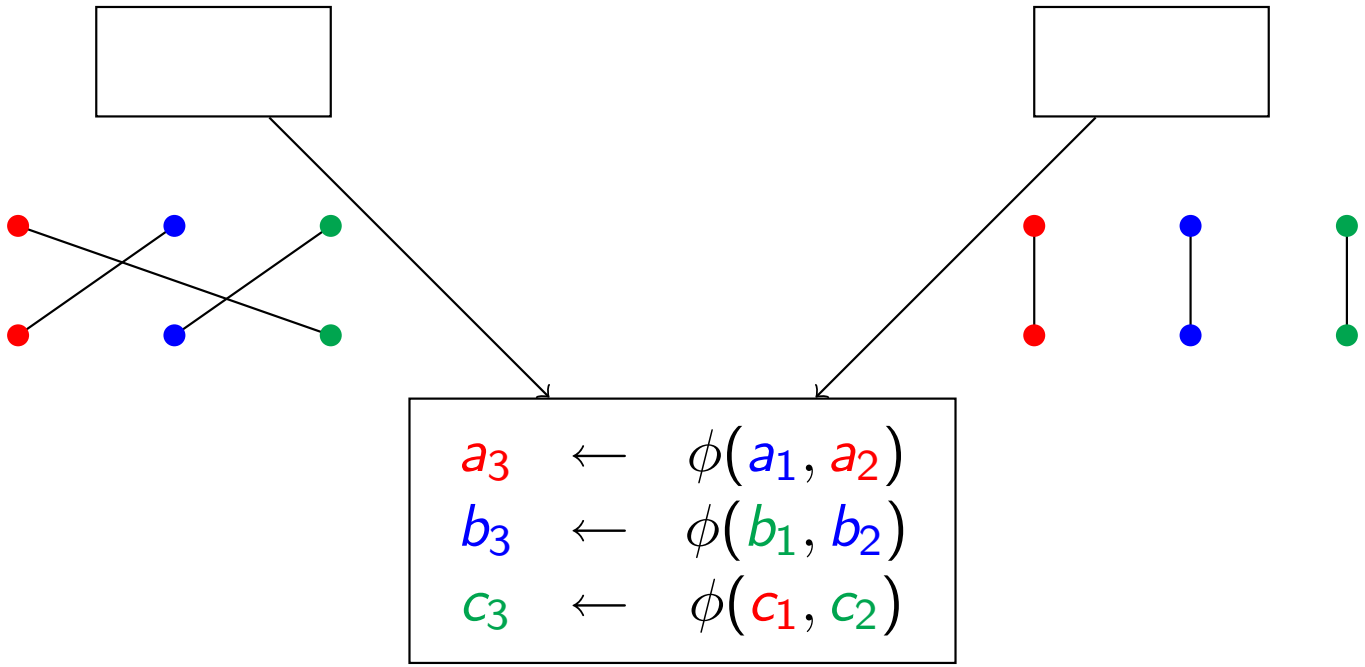
$$\begin{aligned} y_1 &\leftarrow \phi(x_{11}, \ldots, x_{n1}) \\ &\vdots \\ y_m &\leftarrow \phi(x_{1m}, \ldots, x_{nm}) \end{aligned}$$

  execute simultaneously before all other instructions in that block
- Arriving from the i-th edge, the ϕ-functions work as a parallel copy

$$(y_1, \ldots, y_m) \leftarrow (x_{i1}, \ldots, x_{im})$$

ϕ-Functions

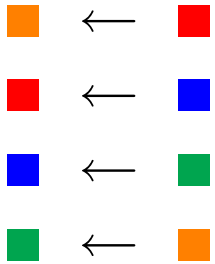
- Consider following example



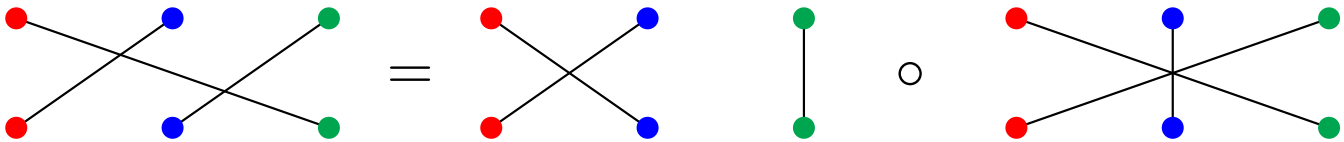
- The ϕs represent register permutations on the control flow edges

Permutations

- A permutation can be implemented with copies if one auxiliary register ■ is available



- Permutations can be implemented by a series of transpositions (i.e. swaps)

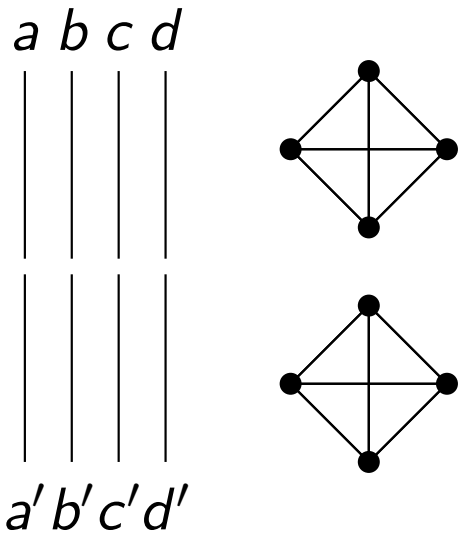


- A transposition can be implemented by three `xors` without a third register

Difference to classical SSA-destruction

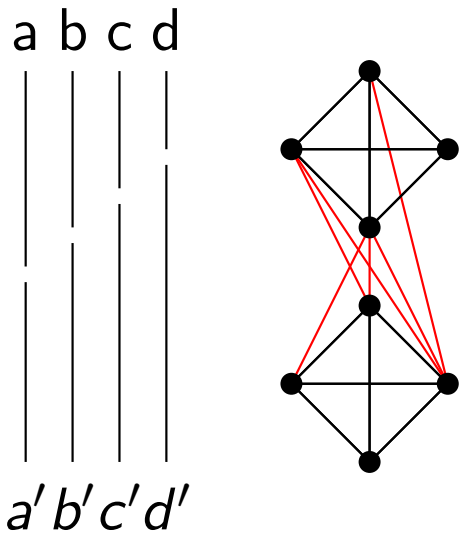
Parallel copies

$$(a', b', c', d') \leftarrow (a, b, c, d)$$



Sequential copies

$$\begin{aligned} d' &\leftarrow d \\ c' &\leftarrow c \\ b' &\leftarrow b \\ a' &\leftarrow a \end{aligned}$$



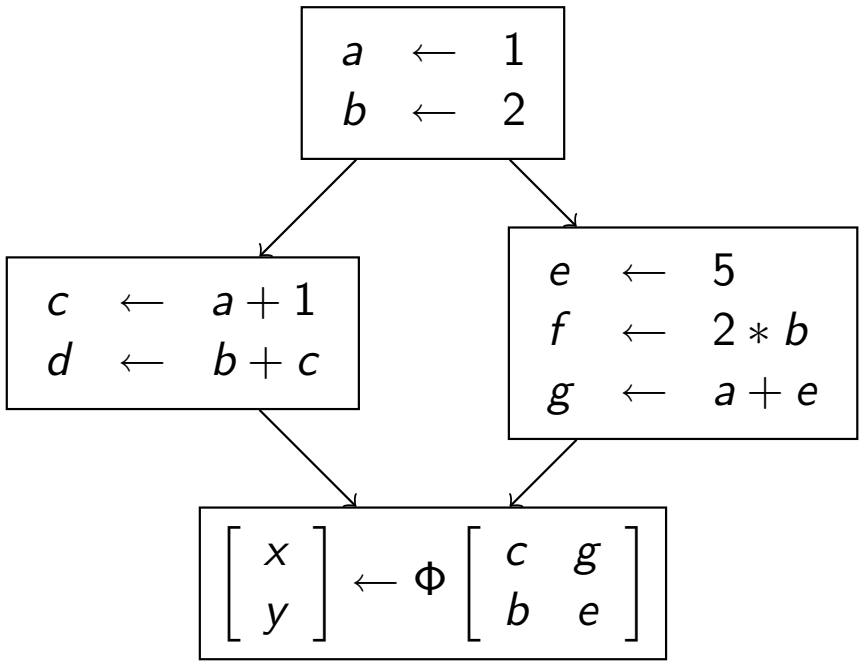
Coalescing

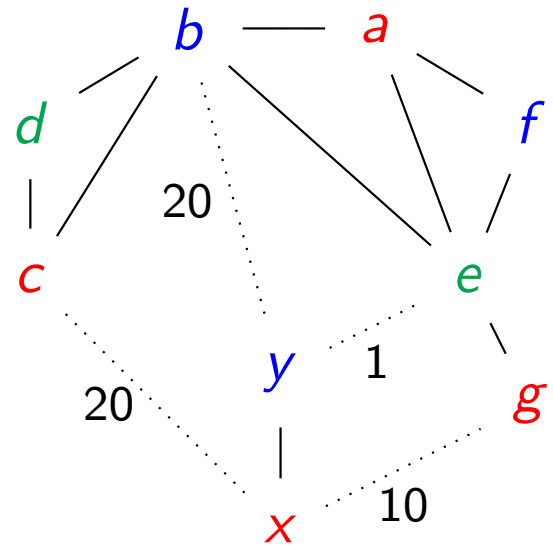
- Minimize number of instructions to be inserted for all ϕs
- Current practice: Merging nodes in the IG to avoid copies
  - Renders the graph unchordal
  - Lose information on the chromatic number
  - When done aggressively, introduce spills in favor of (eliminated) copies

Coalescing

Modeling the Problem

- Given: A minimal coloring of the IG
- Find a feasible coloring with minimal costs
  - Costs are a weighted sum over all equal-color edges
  - Unused colors may be used
  - Structure of graph and program must not be changed





Coalescing

Formal Statement

- Find a k-coloring $\mathcal{C}$ of the IG which assigns as many ϕ-operands and results the same color.

$$\min_{\mathcal{C}} \sum_{\phi} costs(\mathcal{C}, \phi)$$

where

$$costs\left(\mathcal{C}, y \leftarrow \phi(x_1, \ldots, x_n)\right) = \sum_{i=1}^{n} \begin{cases} 0 & \text{if } \mathcal{C}(y) = \mathcal{C}(x_i) \\ w_{yx_i} & \text{else} \end{cases}$$

Coalescing

Solution Strategies

Complexity

- Problem is NP-complete in number of ϕs

Algorithms

- A greedy heuristic
- An optimal method using ILP (integer linear programming)

Coalescing

Heuristic

- Idea: Change colors to achieve more affine pairs
- Problem: Not locally decidable if color change is possible
- Therefore
  - Consider each ϕ seperately
  - Try to give ϕ-operands and result the same color
  - Resolve color clashes recursively through the graph
  - On failure mark the conflict locally and repeat

Coalescing

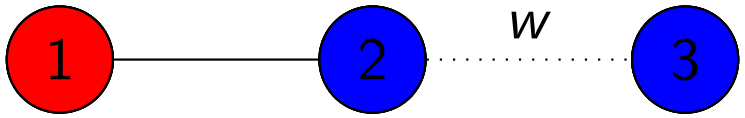
Formalization as ILP

- Binary variables represent states/decisions
- Coloring: $x_{ic} = 1 \Leftrightarrow$ node i has color c
- Optimality: $y_{ij} = 1 \Leftrightarrow$ node i and j have different colors

$$
\begin{array}{lll}
\min f = & \sum_{e \in Q} w_e \cdot y_{ij} & \\
\textit{where} & \sum_{c} x_{ic} = 1 & v_i \in V \\
& x_{ic} + x_{jc} \leq 1 & [v_i, v_j] \in E \\
& y_{ij} \geq x_{ic} - x_{jc} & [v_i, v_j] \in Q \\
\\
& y_{ij}, x_{ic} \in \{0, 1\} &
\end{array}
$$

Coalescing

ILP Example with 3 nodes and 3 colors

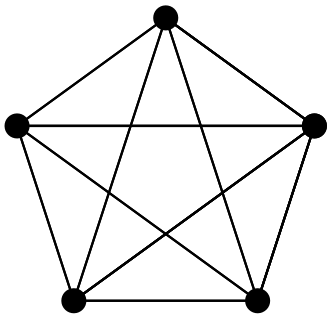


$$
\begin{array}{lll}
\min & ???w \cdot y_{23} & \\
\textit{where} & x_{11} + x_{12} + x_{13} = 1 & \\
 & x_{21} + x_{22} + x_{23} = 1 & \text{coloring} \\
 & x_{21} + x_{22} + x_{23} = 1 & \text{coloring} \\
 & x_{21} + x_{22} + x_{23} = 1 & \text{coloring} \\
 & x_{31} + x_{32} + x_{33} = 1 & \\
 & & \\
\leadsto & x_{11} + x_{21} \leq 1 & \\
 & x_{12} + x_{22} \leq 1 & \text{interference} \\
 & x_{13} + x_{23} \leq 1 & \\
 & x_{11} + x_{21} \leq 1 & \\
 & x_{12} + x_{22} \leq 1 & \text{interference} \\
 & x_{13} + x_{23} \leq 1 &
\end{array}
$$

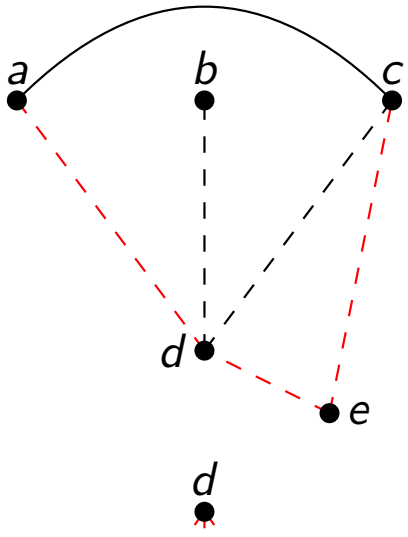
Coalescing

Improving the ILP Runtime

- Clique inequalities
- Path inequalities
- Clique-Path inequalities



Replace $O(n^2)$ inequalities $x_{ic} + x_{jc} \leq 1$ with one: $\sum_{i=1}^{n} x_{ic} \leq 1$



Use incompatibility of interference- and equal-color-edges

$y_{ad} + y_{cd} \geq 1$

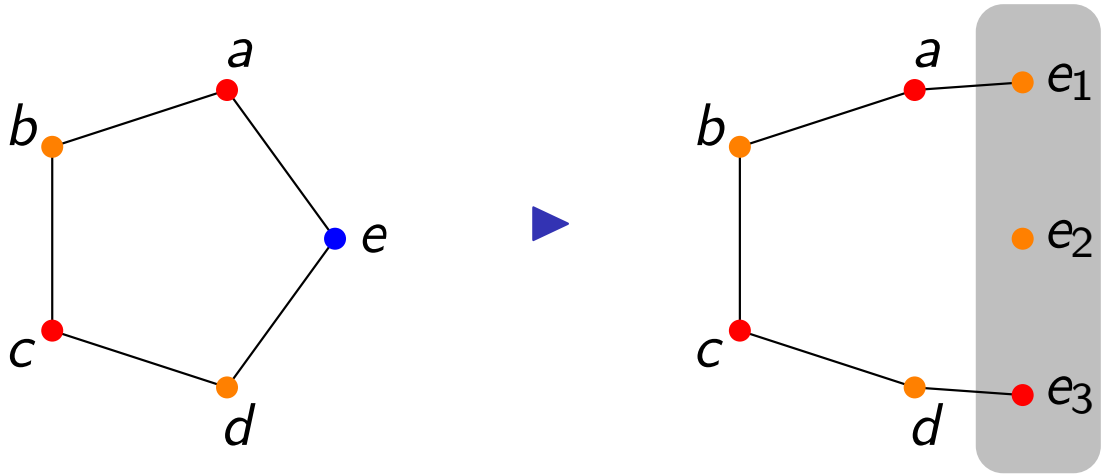
$y_{ad} + y_{de} + y_{ec} \geq 1$

Multiple use of same argument,

Conclusions

SSA vs. non-SSA

- SSA-Construction introduces copies
  (ϕ-operations are copies along edges)
- These copies "blow up" the interference graph
- Nodes are replaced by stable sets

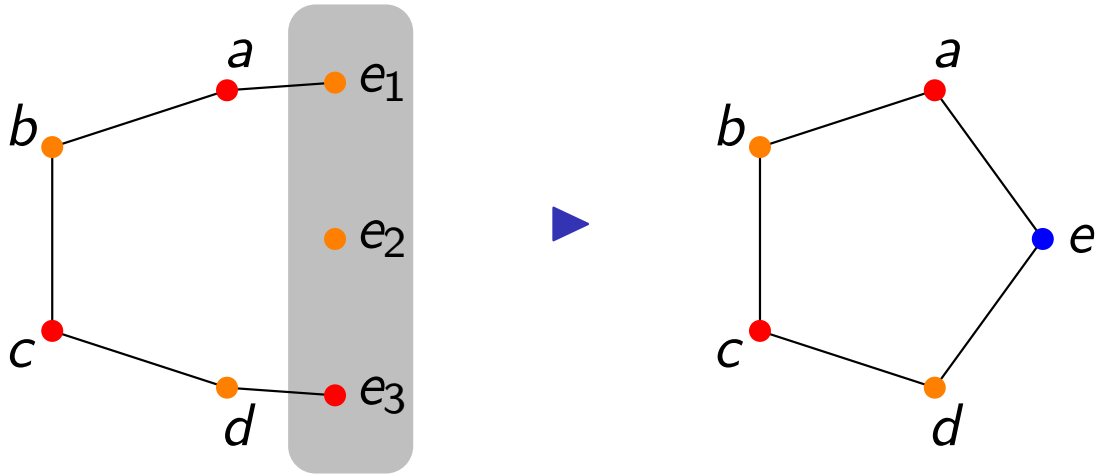


- Breaks cycles in the interference graph
- The interference graph becomes chordal

Conclusions

Non-SSA vs. SSA

- SSA-Destruction coalesces copies aggressively without considering the number of available registers
- Stable sets merged into nodes



- Possibly creating cycles
- Possibly increasing the chromatic number of the graph

Conclusions

- Chordality of SSA IGs allows for decoupling spilling and coalescing
- Register pressure is a precise measure for the number of registers needed
- We know in advance where values must be spilled
  - All labels where the pressure is larger than k
- Spilling can be done before coloring and
- coloring will always succeed afterwards
- Coalescing re-expressed by introducing a cost function on colorings and finding a preferably good coloring

Architecture without iteration